Suggested solutions to the IO (BSc) exam on June 11, 2012
VERSION: June 28, 2012

Question 1a)

- The integrated case.
  - The integrated profit:
  $$\pi_I = (p - c)\, Q(p) = (p - c)(1 - p).$$
  - The p that maximizes π_I:
  $$p_I^*(c) = \frac{1+c}{2}. \quad (1)$$
- Now consider the case where the two firms are not integrated.
  - The retailer chooses p so as to maximize
  $$\pi_R = (p - p_w)\, Q(p) = (p - p_w)(1 - p)$$
  - The p that maximizes π_R:
  $$p^*(p_w) = \frac{1+p_w}{2}. \quad (2)$$
  - Hence, demand for the final good, and therefore for the intermediate good too, is
  $$Q^*(p_w) = 1 - p^*(p_w) = \frac{1-p_w}{2}.$$
  - So the manufacturer's profit as a function of p_w is
  $$\begin{aligned} \pi_M^*(p_w) &= (p_w - c)\, Q^*(p_w) \\ &= (p_w - c)\, \frac{1-p_w}{2}. \quad (3) \end{aligned}$$
  - The p_w that maximizes $\pi_M^*(p_w)$:
  $$p_w = \frac{1+c}{2}. \quad (4)$$
  - Plugging (4) into (2), we have that the retail price under non-integration is
  $$p^*(c) = \frac{3+c}{4},$$
  which is strictly higher than $p_I^*(c)$ in (1).

Question 1b)

- The upstream firm should choose a two-part tariff with the property that it makes the downstream firm face the same effective marginal cost as the integrated firm faces. The latter marginal cost equals c. Therefore, the two-part tariff can be set to

$$T = \begin{cases} A + cq & \text{if } q > 0 \\ 0 & \text{if } q = 0, \end{cases}$$

  where A is a fixed fee that the downstream firm must pay to the upstream firm if wanting to buy any positive quantity, on top of the per-unit price c. As long as A is not too large — so that the downstream firm prefers $q = 0$ — all these two-part tariffs will provide an incentive for the downstream firm to choose the same price as an integrated firm would have chosen (since the effective marginal cost is the same and the demand function of course also is the same).

Question 1c)

- The results under a) and b) suggest that not only the firms but *also the consumers* can gain if two vertically related firms merge (the result under a) or if the upstream firm exerts more control over the downstream firm by using a two-part tariff instead of a linear price (the result under b).

- The policy implication of this (if we take the results seriously) is that we may want to be more inclined to allow mergers between vertically related firms than between horizontally related firms, and we may want to allow upstream firms to control the pricing behavior of the downstream firms (for example by the use of a two-part tariff).

- *From the lecture slides:*

What have we learned from this?

- The interpretation of the above results goes as follows.

- Suppose that two firms that are vertically related want to merge or, for example, the upstream firm wants to impose an RPM clause in its contract with the downstream firm.
  - Then this is not necessarily anti-competitive and harmful to the consumers — all parties may gain.
  - The real problem is the fact that the upstream and downstream firms have monopoly power (or, more generally, market power). The integration/RPM is just a (welfare-enhancing) by-product of that monopoly power.
  - When designing competition policy one may (if one buys the Chicago argument) want to treat vertical relationships differently from horizontal ones. In particular, the argument suggests that interaction between vertically related firms is much more likely to desirable from a welfare point of view.

Question 2a)

- First derive the hot-dog demand from the group of people who have lit up (which all must be "smokers"); denote this demand function by $Q_s(p_s)$. Smokers who have lit up purchase a hot dog if and only if their reservation price (weakly) exceeds the price they must pay:

$$r_s - p_s \geq 0 \Leftrightarrow 2x \geq p_s \Leftrightarrow x \geq \frac{p_s}{2}.$$

  It is assumed that the x values are uniformly distributed on $[0,1]$ and that the total number (or measure) of potential customers in the economy equals one. Therefore, the fraction of people among those who have lit up that purchase a hot dog is given by $1 - \frac{p_s}{2}$ if $p_s \leq 2$, and zero otherwise. Moreover, the fraction of people that (are believed to) have lit up equals $\widehat{\lambda}\gamma$, as there are γ smokers and a fraction $\widehat{\lambda}$ of these are believed to have lit up. The demand from the group of people who have lit up therefore equals

$$Q_s^{light}(p_s) = \begin{cases} \widehat{\lambda}\gamma\left(1 - \frac{p_s}{2}\right) & \text{if } p_s \in [0,2] \\ 0 & \text{if } p_s > 2. \end{cases}$$

  The firm's profits from this group can thus be written as

$$\pi_s(p_s) = p_s Q_s(p_s) = \begin{cases} \widehat{\lambda}\gamma\left(1 - \frac{p_s}{2}\right)p_s & \text{if } p_s \in [0,2] \\ 0 & \text{if } p_s > 2. \end{cases}.$$

  Standard calculations give us the result that these profits are maximized at

$$\boxed{p_s = 1.}$$

- Next derive the hot-dog demand from the group of people who have not lit up (which include both "smokers" and "non-smokers"). From the analysis above it follows that smokers who have not lit up (and who therefore face the price p_n) purchase a hot dog iff $x \geq \frac{p_n}{2}$. The fraction of people belonging to this group equals $\left(1 - \widehat{\lambda}\right)\gamma$. Therefore, given our assumption that the x values are uniformly distributed on $[0,1]$, the demand from the group of smokers who did not light up equals

$$Q_s^{nl}(p_n) = \begin{cases} \left(1 - \widehat{\lambda}\right)\gamma\left(1 - \frac{p_n}{2}\right) & \text{if } p_n \in [0,2] \\ 0 & \text{if } p_n > 2. \end{cases}$$

  Non-smokers purchase a hot dog if and only if their reservation price (weakly) exceeds the price they must pay:

$$r_n - p_n \geq 0 \Leftrightarrow x \geq p_n.$$

  Again using the assumptions that the x values are uniformly distributed on $[0,1]$ and that the fraction of non-smokers is $1-\gamma$, we have that demand from this sub-group is

$$Q_n(p_n) = \begin{cases} (1-\gamma)(1-p_n) & \text{if } p_n \in [0,1] \\ 0 & \text{if } p_n > 1. \end{cases}$$

Adding up the last two demand functions we obtain the result that aggregate demand from those who did not light up a cigarette is

$$Q_{agg}^{nl}\left(p_n\right)=\begin{cases}\left(1-\gamma\right)\left(1-p_n\right)+\gamma\left(1-\widehat{\lambda}\right)\left(1-\frac{p_n}{2}\right) & \text{if } p_n\in[0,1]\\ \gamma\left(1-\widehat{\lambda}\right)\left(1-\frac{p_n}{2}\right) & \text{if } p_n\in[1,2]\\ 0 & \text{if } p_s>2.\end{cases}$$

The firm's profits from this group can thus be written as

$$\begin{aligned}\pi_n\left(p_n\right) &= Q_{agg}^{nl}\left(p_n\right)p_n\\ &= \begin{cases}\left[\left(1-\gamma\right)\left(1-p_n\right)+\gamma\left(1-\widehat{\lambda}\right)\left(1-\frac{p_n}{2}\right)\right]p_n & \text{if } p_n\in[0,1]\\ \gamma\left(1-\widehat{\lambda}\right)\left(1-\frac{p_n}{2}\right)p_n & \text{if } p_n\in[1,2]\\ 0 & \text{if } p_s>2.\end{cases}\end{aligned} \tag{*}$$

- By inspection, this profit expression is continuous in p_n, although its graph may have one or two kinks. Standard calculations give us the result that the profit-maximizing price in the range $p_n\in[1,2]$ equals $p_n=1$. Also note that the derivative of the top line in (*) equals

$$\frac{\partial\pi_n\left(p_n\right)}{\partial p_n}=\left(1-\gamma\right)\left(1-2p_n\right)+\gamma\left(1-\widehat{\lambda}\right)\left(1-p_n\right). \tag{**}$$

  Evaluated at $p_n=1$, this derivative is negative. This means that the graph of the overall function looks (in qualitative terms) as in the attached figure. In particular, the optimal price must lie in the region where $p_n<1$. Setting the derivative in (**) equal to zero and solving the resulting expression for p_n, we have that the profit-maximizing price equals

$$\boxed{p_n=\frac{1-\gamma\widehat{\lambda}}{2-\gamma\left(1+\widehat{\lambda}\right)}\in(0,1).}$$

Question 2b)

- The players optimal behavior at stages (ii) and (iii) has already been characterized in the a) question, and it is summarized by the two optimally chosen prices p_s and p_n, which are both stated in the a) question.

- In order to solve the overall game and find the fraction λ^*, the next step would be to consider the smokers' optimal behavior at stage (i). There they decide whether to light up a cigarette or not. It is already explained in the question that a smoker will light up if and only if the inequality at the bottom of page 2 is satisfied.[1] We must evaluate the inequality at the prices derived in a), for the smokers are forward-looking and understand what prices that will be set later, given the salesman's beliefs $\widehat{\lambda}$. A smoker who is *indifferent* between lighting up and not will have a b value that makes the inequality hold as an equality. Denote this b value by $b\left(\widehat{\lambda}\right)$.

[1] In this inequality there is both a p_s and a p_s^*. This is a typo. The two pieces of notation are supposed to be the same (and given that this equation is not yet evaluated at the equilibrium of the overall game, my preferred notation here is p_s). Therefore, simply ignore the asterisk in p_s^*. Similarly with p_n and p_n^*.

- The smokers who will decide to light up are the ones with a b value larger than or equal to $b\left(\widehat{\lambda}\right)$. Given our assumption about a uniform distribution, these smokers make up the fraction $1-b\left(\widehat{\lambda}\right)$ of all the smokers. That is, we must have $\lambda = 1 - b\left(\widehat{\lambda}\right)$, where λ is the fraction of smokers that light up. Moreover, at an equilibrium the beliefs must be correct, which means that $\widehat{\lambda} = \lambda = \lambda^*$. We can therefore characterize the equilibrium fraction of smokers that light up by the equation $\lambda^* = 1 - b(\lambda^*)$. This equation could in principle be solved explicitly for the roots λ^*, although in this model it would be messy to do this as the equation is a third-degree polynomial.

Question 2c)

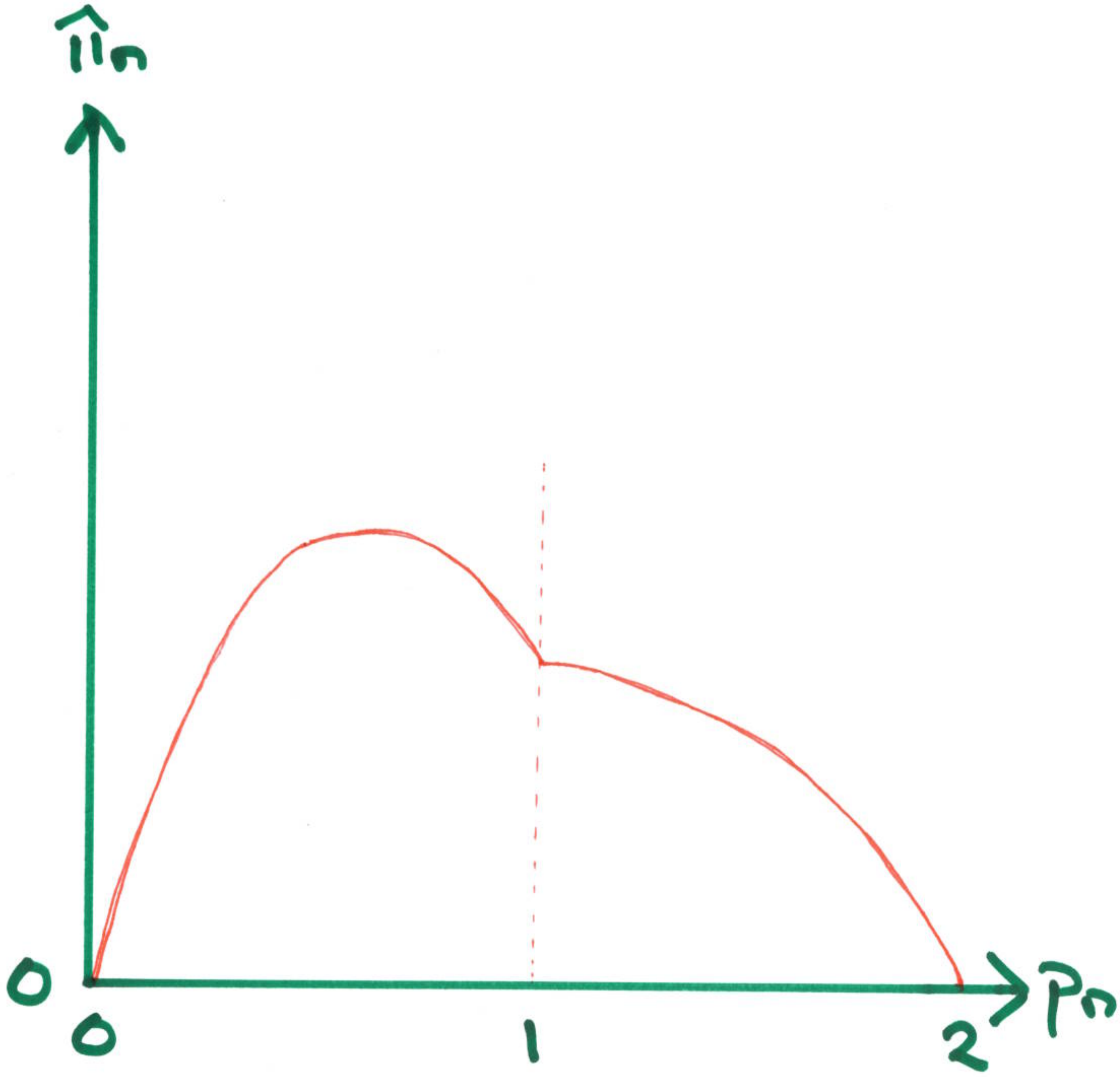
- *Term 1:* This represents the utility that those smokers who light up receive from smoking. There are γ smokers all together (which is why this parameter shows up in front of the integration sign), and those who light up are the ones with b values ranging from $1-\lambda^*$ up to one. The utility that each one of those smokers obtain from smoking equals b, which is why we integrate over b.
- *Term 2:* This term represents the utility that those smokers who light up and who buy a hot dog up receive from consuming the hot dog. There are $\gamma\lambda^*$ smokers who light up, and the ones of these with an x value exceeding $p_s/2$ will buy a hot dog. The net surplus of each of those who buy a hot dog is $2x - p_s$.
- *Term 3:* This term represents the utility that those smokers who *do not* light up and who buy a hot dog up receive from consuming the hot dog. It is analogous to term 2, except that these customers must pay the price p_n, and the number of smokers who do not light up is $\gamma(1-\lambda^*)$.
- *Term 4:* This term represents the utility that the non-smokers who buy a hot dog up receive from consuming the hot dog. It is analogous to terms 2 and 3, except that the number of non-smokers is $1-\gamma$, the ones with an x value exceeding p_n buy, and their net surplus if buying is $x - p_n$.

Question 2d)

- To answer the question we first must try to understand what the equilibrium behavior would be like if price discrimination was not allowed (this situation is below referred to as "the benchmark"). It is clear that then all smokers would decide to light up, because in this model the benefit of lighting up is $b \geq 0$ and there is no cost of lighting up if this cannot lead to a higher hot-dog price.
- We should also expect that the (single) hot-dog price in the benchmark would lie in between the prices p_s^* and p_n^*.
  - For the equilibrium price should be higher the higher is the relative number of high-demand customers (i.e., smokers) in the aggregate demand. Moreover, the demand that gives rise to p_s^* includes *only*

high-demand customers (and no low-demand customers at all); and the demand that gives rise to p_n^* has some of the high-demand customers and all of the low-demand customers. The benchmark case gives rise to an intermediate case as this demand includes all customers of each type.

- There are three categories of consumers that we must distinguish:
  - *Smokers who lit up in the model with price discrimination:* These consumers will get the same utility from smoking in the benchmark as they get in the model with price discrimination. However, they will have to pay a lower price in the benchmark. Members of this category therefore unambiguously gain from the law.
  - *Smokers who did not light up in the model with price discrimination:* These consumers will indeed light up in the benchmark and therefore get utility from smoking, which they did not get in the model with price discrimination. However, in the benchmark they will also have to pay a higher price for the hot dog. Therefore, whether members of this category gain or lose from the law depends on which one of these two effects is the strongest.
  - *Non-smokers:* These consumers never light up, so whether they gain or lose depends only on the change in the hot-dog price. Since the hot-dog price that these consumers must pay goes up, they will unambiguously lose from the introduction of the law.



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June 2012, fig